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Some Topological Indices in Neutrosophic Graphs

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ABSTRACT

Neutrosophic Graphs are graphs that follow three-valued logic and may be considered a fuzzy graph, in some cases, it is difficult to optimize and model using fuzzy graphs. In this paper, the first and second Zagreb indices, the Harmonic index, the Randic' index and the Connectivity index for these graphs are investigated and some of the theorems related to these indices are discussed and proven. These indices are also calculated for some specific types of Neutrosophic Graphs, such as regular Neutrosophic Graphs.

KEYWORDS: Neutrosophic Graphs, Zagreb indices, Harmonic index, Randic' index, Connectivity index

1 INTRODUCTION

Graph theory has many real applications for modeling problems in various computer applications, systems analysis, computer networks, transportation, operations research and economics. Graphs are essentially a pattern of relationships, and they are used to illustrate the real-life problem of relationships between objects. The vertices and edges of the graph are used to represent objects and the relationships between objects, respectively. Many of the optimization issues are caused by inaccurate information for various reasons, including loss of information, lack of evidence, incomplete statistical data, and lack of sufficient information. And this creates uncertainty in various issues. There can usually be uncertainty about real-life issues in the information that defines the problem. Classical Graphic Theory uses the basic concept of classical set theory, as proposed by Contour. In a classic graph, for each vertex or edge, there are two possibilities: either in the graph or not in the graph. Therefore, classical graphs cannot model uncertain optimization problems. Real-life issues are often unclear, making modeling using classical graphs difficult. Fuzzy set [1] is a generalized version of the classical set in which objects have different membership degrees. A fuzzy set gives the degree of different members between zero and one. Much work has already been done on fuzzy graphs, including the calculation of various topology indices. Indicators such as Zagreb index, Randic', harmonic and so on. But there is another class of graphs that is a broad case of fuzzy graphs. In this type of graphs, known as neutrosophic graphs, in addition to the degree of accuracy of each membership function, the degree of its membership is uncertain, as well as its inaccuracy. So in many cases it may be more logical to use this model than graphs in real-world problems. Therefore, in this paper we try to calculate for the first time some topological indices for this type of graph.

2 PRELIMINARIES

In this section, provides some definitions and theorems needed.

Definition 1. [2] Let $G = (N, M)$ is an single-valued Neutrosophic graph, where N is a Neutrosophic set on V and, M is a Neutrosophic set on E, which satisfy the following

$$T_M(u,v) \leq min\big(T_N(u), T_N(v)\big),$$
$$I_M(u,v) \geq max\big(I_N(u), I_N(v)\big),$$
$$F_M(u,v) \geq max\big(F_N(u), F_N(v)\big),$$

Where u and v are two vertices of G, and $(u,v) \in E$ is an edge of G.

Definition 2. [2] Given $G = (N, M)$ is a single-valued neutrosophic graph and P is a path in G. P is a collection of different vertices, $v_0, v_1, v_2, \dots, v_n$ such that $\big(T_M(v_{i-1}, v_i), I_M(v_{i-1}, v_i), F_M(v_{i-1}, v_i)\big) > 0$ for $0 \leq i \leq n$. P is a neutrosophic cycle if $v_0 = v_n$ and $n \geq 3$.

Definition 3. [2] Given $G = (N, M)$ is a single-valued neutrosophic graph. G is a connected single-valued Neutrosophic graph if there exists no isolated vertex in G. ($v \in V_G$ is isolated vertex, if there exists no incident edge to the vertex v.)

Definition 4. [2] Given $G = (N, M)$ is a single-valued neutrosophic graph, and $v \in V$ is vertex of G. the degree of vertex v is the sum of the truth membership values, the sum of the indeterminacy membership values, and the sum of the falsity membership values of all the edges that are adjacent to vertex v. and is denoted by $d(v)$, that

$$d(v) = \big(d_T(v), d_I(v), d_F(v)\big) = \left(\sum_{\substack{v \in V \\ v \neq u}} T_M(v,u), \sum_{\substack{v \in V \\ v \neq u}} I_M(v,u), \sum_{\substack{v \in V \\ v \neq u}} F_M(v,u)\right).$$

Definition 5. [2] Given $G = (N, M)$ is a single-valued neutrosophic graph, and the d_m–degree of any vertex v in G is denoted as $d_m(v)$ where

$$d_m(v) = \left(\sum_{u \neq v \in V} T_M^m(u,v), \sum_{u \neq v \in V} I_M^m(u,v), \sum_{u \neq v \in V} F_M^m(u,v)\right)$$

Here, the path $v = v_0, v_1, v_2, \dots, v_n = u$ is the shortest path between the vertices v and u, when the length of this path is m.

Definition 6. [2] Given $G = (N, M)$ is a single-valued neutrosophic graph, G is a regular neutrosophic graph if it satisfies the following,

$$\sum_{v \neq u} T_M(v,u) = c, \sum_{v \neq u} I_M(v,u) = c, \sum_{v \neq u} F_M(v,u) = c,$$

Where c is a constant value.

3 TOPOLOGICAL INDICES IN NEUTROSOPHIC GRAPHS

3-1 Zagreb index of First and Second Kind in Neutrosophic Graphs

Definition 8. Let $G = (N, M)$ be the neutrosophic graph whit non-empty vertex set. The first Zagreb index is denoted by $M(G)$ and defined as

$$M(G) = \sum_{i=1}^{n} (T_N(u_i), I_N(u_i), F_N(u_i)) d_2(u_i), \qquad \forall\ u_i \in\ V.$$

Example 1. Let $G = (N, M)$ be the neutrosophic graph with $V = \{a, b, c\}$ where $(T_N, I_N, F_N)(a) = (0.3, 0.6, 0.7), (T_N, I_N, F_N)(b) = (0.3, 0.5, 0.6),$ and $(T_N, I_N, F_N)(c) = (0.4, 0.5, 0.6),$ The edge set contains $(T_M, I_M, F_M)(a, b) = (0.2, 0.6, 0.8),$ $(T_M, I_M, F_M)(b, c) = (0.2, 0.6, 0.7),$ and $(T_M, I_M, F_M)(a, c) = (02, 0.8, 0.9).$

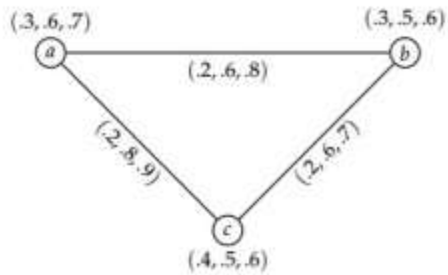

Figure 1. a neutrosophic graph with $V = \{a, b, c\}$

The first Zagreb index is

$$d(a) = (0.2 + 0.2, 0.6 + 0.8, 0.8 + 0.9) = (0.4, 1.4, 1.7),$$
$$d(b) = (0.2 + 0.2, 0.6 + 0.6, 0.8 + 0.7) = (0.4, 1.2, 1.5),$$
$$d(c) = (0.2 + 0.2, 0.8 + 0.6, 0.9 + 0.7) = (0.4, 1.4, 1.6).$$

Now, we have

$$d_2(a) = (0.04 + 0.04, 0.36 + 0.64, 0.64 + 0.81) = (0.08, 1, 1.45),$$
$$d_2(a) = (0.04 + 0.04, 0.36 + 0.36, 0.64 + 0.49) = (0.08, 0.72, 1.13),$$
$$d_2(a) = (0.04 + 0.04, 0.64 + 0.36, 0.81 + 0.49) = (0.08, 1, 1.3).$$

$$\begin{aligned} M(G) = \sum_{i=1}^{4} & (T_N(u_i), I_N(u_i), F_N(u_i)) d_2(u_i) \\ &= (0.3, 0.6, 0.7)(0.08, 1, 1.45) + (0.3, 0.5, 0.6)(0.08, 0.72, 1.13) \\ &+ (0.4, 0.5, 0.6)(0.08, 1.1.3) \\ &= (0.024 + 0.6 + 1.015) + (0.024 + 0.36 + 0.678) + (0.032 + 0.5 + 0.78) \\ &= 4.013. \end{aligned}$$

Definition 9. The second Zagreb index is denoted by $M^*(G)$ and defined as

$$M^*(G) = \frac{1}{2} \sum [(T_N(u_i), I_N(u_i), F_N(u_i)) d(u_i)][(T_N(v_j), I_N(v_j), F_N(v_j)) d(v_j)]$$
$$\forall i \neq j \ and \ (u_i, v_j) \in E.$$

Example 2. If G is the same neutrosophic graph as example 1, we have

$$M^*(G) = \frac{1}{2}[(0.3, 0.6, 0.7).(0.4, 1.4, 1.7) \times (0.3, 0.5, 0.6).(0.4, 1.2, 1.5)$$
$$+ (0.3, 0.6, 0.7).(0.4, 1.4, 1.7) \times (0.4, 0.5, 0.6).(0.4, 1.4, 1.6)$$
$$+ (0.3, 0.5, 0.6).(0.4, 1.2, 1.5) \times (0.4, 0.5, 0.6).(0.4, 1.4, 1.6)]$$
$$= \frac{1}{2}[(0.12 + 0.84 + 1.19) \times (0.12 + 0.6 + 0.9) + (0.12 + 0.84 + 1.19)$$
$$\times (0.16 + 0.7 + 0.96) + (0.12 + 0.6 + 0.9) \times (0.16 + 0.7 + 0.96)$$
$$= \frac{1}{2}[(2.15)(1.62) + (2.15)(1.82) + (1.62)(1.82)] = \frac{1}{2}(10.3444) = 5.1722.$$

Note 1. As we have seen, the value of $M^*(G)$ is less than the value of $M(G)$, and this is always the case.

Theorem 1. Let G is the neutrosophic graph and H is the neutrosophic sub graph of G such that $H = G - u$ then $M(H) < M(G)$ and $M^*(H) < M^*(G)$.

Proof. Given that by omitting a vertex of G, a positive value, the sum is lost, so the proof is obvious.

□

3-2 Harmonic index in Neutrosophic Graphs

Definition 10. The Harmonic index of neutrosophic graph G is defined as

$$H(G) = \sum \frac{1}{(T_N(u_i), I_N(u_i), F_N(u_i))d(u_i) + (T_N(v_j), I_N(v_j), F_N(v_j))d(v_j)},$$

$$\forall i \neq j \text{ and } (u_i, v_j) \in E.$$

Example 3. We have the previous example,

$$H(G) = \frac{1}{(0.3, 0.6, 0.7)(0.4, 1.4, 1.7) + (0.3, 0.5, 0.6)(0.4, 1.2, 1.5)}$$
$$+ \frac{1}{(0.3, 0.6, 0.7)(0.4, 1.4, 1.7) + (0.4, 0.5, 0.6)(0.4, 1.4, 1.6)}$$
$$+ \frac{1}{(0.3, 0.5, 0.6)(0.4, 1.2, 1.5) + (0.4, 0.5, 0.6)(0.4, 1.4, 1.6)}$$
$$= \frac{1}{2.15 + 1.62} + \frac{1}{2.15 + 1.82} + \frac{1}{1.62 + 1.82} = \frac{1}{3.77} + \frac{1}{3.97} + \frac{1}{3.44} = 0.8078.$$

3-3 Randic' index in Neutrosophic Graphs

Definition 11. The Randic' index of neutrosophic graph G is defined as

$$R(G) = \sum \left((T_N(u_i), I_N(u_i), F_N(u_i))d(u_i) \left(T_N(v_j), I_N(v_j), F_N(v_j)\right) d(v_j) \right)^{\frac{-1}{2}},$$
$$\forall\ i \neq j \text{ and } (u_i, v_j) \in E$$

Example 3. For above example,

$$R(G) = \frac{1}{\sqrt{(0.3, 0.6, 0.7).(0.4, 1.4, 1.7) \times (0.3, 0.5, 0.6).(0.4, 1.2, 1.5)}} + \frac{1}{\sqrt{(0.3, 0.6, 0.7).(0.4, 1.4, 1.7) \times (0.4, 0.5, 0.6).(0.4, 1.4, 1.6)}} + \frac{1}{\sqrt{(0.3, 0.5, 0.6).(0.4, 1.2, 1.5) \times (0.4, 0.5, 0.6).(0.4, 1.4, 1.6)}}$$
$$= \frac{1}{\sqrt{2.15 \times 1.62}} + \frac{1}{\sqrt{2.15 \times 1.82}} + \frac{1}{\sqrt{1.62 \times 1.82}} = 1.6237.$$

3-4 Connectivity index in Neutrosophic Graphs

Definition 12. Let $G = (N, M)$ be the neutrosophic graph. The connectivity index of G is defined by

$$CI(G) = \sum_{u_i, v_j \in V} (T_N(u_i), I_N(u_i), F_N(u_i))(T_N(v_j), I_N(v_j), F_N(v_j)) \times (T_{CONN\,G} + I_{CONN\,G} + F_{CONN\,G})(u_i, v_j).$$

Where $CONN_G(u_i, v_j)$ is the strength of connectedness between u_i and v_j.

Definition 13. The strength of connectedness between u_i and v_j is defined as sum on the components of maximum of strengths of all paths between u_i and v_j.

For example, in the above figure, the strength of connectedness between:
a and b is $CONN_G(a, b) = \max\{(0.2, 0.6, 0.8), (0.2, 0.6, 0.7)\} = (0.2, 0.6, 0.8)$,
a and c is $CONN_G(a, c) = \max\{(0.2, 0.8, 0.9), (0.2, 0.6, 0.7)\} = (0.2, 0.8, 0.9)$,
b and c is $CONN_G(b, c) = \max\{(0.2, 0.6, 0.7), (0.2, 0.6, 0.8)\} = (0.2, 0.6, 0.8)$.
Then,

$$CI(G) = \sum_{u_i, v_j \in V} (T_N(u_i), I_N(u_i), F_N(u_i))(T_N(v_j), I_N(v_j), F_N(v_j)) \times (T_{CONN\,G} + I_{CONN\,G} + F_{CONN\,G})(u_i, v_j)$$
$$= (0.3, 0.6, 0.7).(0.3, 0.5, 0.6) \times (0.2 + 0.6 + 0.8) + (0.3, 0.6, 0.7).(0.4, 0.5, 0.6) \times (0.2 + 0.8 + 0.9) + (0.3, 0.5, 0.6).(0.4, 0.5, 0.6) \times (0.2 + 0.6 + 0.8)$$
$$= (0.09 + 0.3 + 0.42)(1.6) + (0.12 + 0.3 + 0.42)(1.9) + (0.12 + 0.25 + 0.36)(1.6) = (0.81)(1.6) + (0.84)(1.9) + (0.73)(1.6)$$
$$= 4.06.$$

Theorem 2. Let G and H be the two neutrosophic graphs are isomorphic, then the topological indices values of two neutrosophic graphs are equal.

Proof. To prove, consider $G = (V_G, N_G, M_G)$ and $H = (V_H, N_H, M_H)$ be isomorphic Neutrosophic Graphs. Hence there is an identity function $\mu_N: N_G(u) \to N_H(u^*)$, for all $u \in V_G$ there exist $u^* \in V_H$ as well as $\mu_M: M_G(u, v) \to M_H(u^*, v^*)$, then each vertex of G corresponds to an vertex in H, with the same membership value and the same edges. Hence, the neutrosophic graph structure may differ but collection of vertices and edges are same gives the equal topological indices value.

□

Theorem 3. Let $G = (V_G, N_G, M_G)$, is a neutrosophic graph and H is the neutrosophic sub graph of G, So that H is made by removing edge $uv \in M_G$ from G. Then, we have, $CI(H) < CI(G)$ iff uv is a bridge.

Proof. To prove the first side of the theorem we consider two cases:

Case 1. Let uv be an edge with all three components having the least value, so the edge uv will have no effect on the result. So we have $CI(H) = CI(G)$.

Case 2. Now suppose that uv is an edge that has maximum components, so they will have an effect on $CONN_G(u,v)$. So by removing edge uv, the value of $CONN_G(u,v)$ will decrease, then we have $CI(H) < CI(G)$. Since the bridge is called the edge that has its deletion reducing the $CONN_G$, so, uv is a bridge.

Conversely, given that uv is a bridge. According to the definition of bridge we have, $CONN_G(u,v) > CONN_{G-uv}(u,v)$, So we conclude that, $CI(H) < CI(G)$.

□

4 CONCLUSION

In this paper, for the first time, some topological indices for neutrosophic graphs are defined. This topic has a lot of work to do, and it can also be used for its results on various issues related to this category of graphs.

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